Nuclear Physics on the Light Front–a new old way to do an old new problem

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Abstract. A brief introduction to light front techniques is presented. This is followed by a review of recent attempts to perform realistic, relativistic nuclear physics with those techniques.

MOTIVATION

This lecture series is aimed at describing our recent attempts to derive the properties of nuclei using the light-front formalism. Nuclear properties are very well handled within existing conventional nuclear theory, so it is necessary to explain the motivation. It seems to me that understanding experiments involving high energy nuclear reactions requires that light-front dynamics and light cone variables be used. Consider the EMC experiment [1], which showed that there is a significant difference between the parton distributions of free nucleons and nucleons in a nucleus. This difference can interpreted as a shift in the momentum distribution of valence quarks towards smaller values of the Bjorken variable x. This variable is a ratio of the plus-momentum $k^+ = k^0 + k^3$ of a quark to that of the target. If one uses k^+ as a momentum variable, the corresponding canonical spatial variable is $x^- = x^0 - x^3$ and the time variable is $x^+ = x^0 + x^3$. To do calculations in this framework is to use light front dynamics.

Light front dynamics applies to nucleons within the nucleus as well as to partons of the nucleons, and this is a useful approach whenever the momentum of initial or final state nucleons is large compared to their mass [2]. For example, this technique can be used for $(e, e'p)$ and $(p, 2p)$ reactions at sufficiently high energies. The use of light-front variables for nucleons in a nucleus is not sufficient. It is also necessary to include all the relevant features of conventional nuclear dynamics. Combining these two aspects provides the technical challenge which we have been addressing.

[1]) This work is partially supported by the USDOE

I'd like to begin by describing how using the light-front approach leads to important simplifications. Consider high energy electron scattering from nucleons in nuclei. Let the four-momentum q of the exchanged virtual photon be given by $\left(\nu, 0, 0, -\sqrt{Q^2+\nu^2}\right)$, with $Q^2 = -q^2$, and Q^2 and ν^2 are both very large but Q^2/ν is finite (the Bjorken limit). Use the light-cone variables $q^\pm = q^0 \pm q^3$ in which $q^+ \approx Q^2/2\nu = Mx$, $q^- \approx 2\nu - Q^2/2\nu$, so that $q^- \gg q^+$. Here M is the mass of a nucleon. We neglect q^- in comparison to q^+; corrections to this can be handled in a systematic fashion. Then the scattering cross section for $e + A \rightarrow e' + (A-1)_f + p$, where f represents the final nuclear eigenstate of P^-, and p the four-momentum of the final proton, takes the form

$$d\sigma \sim \sum_f \int \frac{d^3 p_f}{E_f} \int d^4 p\, \delta(p^2 - M^2) \delta^{(4)}(q + p_i - p_f - p) |\langle p, f \mid J(q) \mid i \rangle|^2, \tag{1}$$

with the operator $J(q)$ as a schematic representation of the electromagnetic current. Performing the four-dimensional integral over p leads to the expression

$$d\sigma \sim \sum_f \int \frac{d^2 p_f d p_f^+}{p_f^+} \delta\left((p_i - p_f + q)^2 - m^2\right) \mid \langle p, f \mid J(q) \mid i \rangle \mid^2 . \tag{2}$$

The argument of the delta function $(p_i - p_f + q)^2 - M^2 \approx -Q^2 + 2q^-(p_i - p_f)^+$. Thus p_f^- does not appear in the argument of the delta function, or anywhere else, so that we can replace the sum over intermediate states by unity. In the usual equal-time representation, the argument of the delta function is $-Q^2 + 2\nu(E_i - E_f)$. The energy of the final state appears, and one can not do the sum over states. It is useful to define $\boldsymbol{p_B} \equiv \boldsymbol{p_i} - \boldsymbol{p_f}$, because $p_B^+ = Q^2/2\nu \equiv Mx$ (as demanded by the delta function). Then one can re-express Eq. (2) as

$$d\sigma \sim d^2 p_{B\perp} n(Mx, p_{B\perp}), \tag{3}$$

where $n(Mx, p_{B\perp})$ is the probability for a nucleon in the ground state to have a momentum $(Mx, p_{B\perp})$. Integration in Eq. (3) leads to

$$\sigma \sim \int d^2 p_\perp\, n(Mx, p_\perp) \equiv f(Mx), \tag{4}$$

with $f(Mx)$ as the probability for a nucleon in the ground state to have a plus momentum of Mx. The use of light-front dynamics to compute nuclear wave functions should allow us to compute $f(Mx)$ from first principles. We also claim that using light-front dynamics incorporates the experimentally relevant kinematics from the beginning, and therefore is the most efficient way to compute the cross sections for nuclear deep inelastic scattering and nuclear quasi-elastic scattering.

Since much of this work is motivated by the desire to understand nuclear deep inelastic scattering and related experiments, it is worthwhile to review some of the features of the EMC effect [1,3]. One key experimental result is the suppression of

the structure function for $x \sim 0.5$. This means that the valence quarks of bound nucleons carry less plus-momentum than those of free nucleons. This may be understood by postulating that mesons carry a larger fraction of the plus-momentum in the nucleus than in free space. While such a model explains the shift in the valence distribution, one obtains at the same time a meson (i.e. anti-quark) distribution in the nucleus, which is strongly enhanced compared to free nucleons and which should be observable in Drell-Yan experiments [4]. However, no such enhancement has been observed experimentally [5], and the implications are analyzed in Ref. [6].

The use of light-front dynamics should allow us to compute the necessary nuclear meson distribution functions using variables which are experimentally relevant. The need for a computation of such functions in a manner consistent with generally known properties of nuclei led us to begin this program. There are other motivations for using the light front formalism that have been emphasized in many reviews [7]. One key feature is that the vacuum of the theory is trivial because it can not create pairs. Another is that the theory is a Hamiltonian theory and the many-body techniques of equal time theory can be used here too. I also like to say: Ask not what the light front can do for nuclear physics; instead ask what nuclear physics can do for the light front. This is to provide a set of non-trivial four dimensional examples with real physics content. Finally I quote the review by Geesaman et al. "In light front dynamics LFD, the particles are on mass-shell, and there are no off-shell ambiguities. However, ... we have little or no experience in calculating the wave function of a realistic nucleus in LFD". The aim here is to provide such wave functions.

Outline

We shall begin with a simple description of what is light front dynamics. Then the formal procedures of light front quantization of a hadronic Lagrangian $\mathcal{L}$ will be discussed. The first application is a study of infinite nuclear matter within the mean field approximation [8]. The distribution functions $f(y)$ for nucleons and mesons will be computed. The above topics comprise the first lecture. The next lecture is devoted to a study of finite nuclei [9] using the mean field approximation. Here one must confront a difficulty. The use of $x^- = t - z$ as a spatial variable violates manifest rotational invariance because x^- and $x_\perp$ are different variables. We show that rotational invariance re-emerges after one does the appropriate dynamical calculation. It is necessary to go beyond the mean field approximation, and the third lecture deals with that [10]. Nucleon-nucleon scattering is studied first and used in the many-body calculation. The influence of nucleon-nucleon correlations on the properties of nuclear matter is studied by making the necessary light front calculations. Applications are to compute the nuclear pionic content and to nuclear deep inelastic scattering and Drell-Yan processes. The goal is to provide a series of examples showing that the light front can be used for high energy realistic and relativistic nuclear physics.

WHAT IS LIGHT FRONT DYNAMICS?

This is a relativistic treatment of dynamics in which the fields are quantized at a fixed "time"$\tau = t + z = x^0 + x^3 \equiv x^+$. This means that the orthogonal spatial variable must be $x^- \equiv t - z$ so that the canonical momentum is $p^0 + p^3 \equiv p^+$. The remainder of the spatial variables are given by: $\vec{x}_\perp, \vec{p}_\perp$.

The consequence of using τ as a "time" variable is that the canonical energy is $p^- = p^0 - p^3$. In general our notation is given by

$$A^\pm \equiv A^0 \pm A^3, \tag{5}$$

with

$$A \cdot B = A^\mu B_\mu = \frac{1}{2}\left(A^+ B^- + A^- B^+\right) - \vec{A}_\perp \cdot \vec{B}_\perp. \tag{6}$$

The key reason for using such unusual coordinates is phenomenological. For a particle with $\vec{v} \approx c\hat{e}_3$, the quantity p^+ is BIG. Thus experiments tend to measure quantities associated with p^+.

Another important feature is the relativistic dispersion relation $p^\mu p_\mu = m^2$, which in light front dynamics takes the form:

$$p^- = \frac{p_\perp^2 + m^2}{p^+}. \tag{7}$$

Thus one has a form of relativistic kinematics that avoids using a square root.

The main formal consequence of using light front dynamics is that the minus component of the total momentum, P^-, is used as a Hamiltonian operator, and the plus component P^+ is used as a momentum operator. The procedures to obtain these operators are discussed in the next section.

LIGHT FRONT QUANTIZATION

My intent here is to discuss the basic aspects in as informal way as possible. For more details see the reviews and the references. I'll start by considering one free field at a time. These will be the scalar meson ϕ, the Dirac fermion ψ and the massive vector meson V^μ.

Free Scalar field

Consider the Lagrangian

$$\mathcal{L}_\phi = \frac{1}{2}(\partial^+\phi\partial^-\phi - \boldsymbol{\nabla}_\perp\phi \cdot \boldsymbol{\nabla}_\perp\phi - m_s^2\phi^2). \tag{8}$$

The notation is such that $\partial^{\pm} = \partial^0 \pm \partial^3 = 2\frac{\partial}{\partial x^{\mp}}$. The Euler-Lagrange equation leads to the wave equation

$$i\partial^- \phi = \frac{-\nabla_\perp^2 + m_s^2}{i\partial^+}\,\phi. \tag{9}$$

The most general solution is a superposition of plane waves:

$$\phi(x) = \int \frac{d^2k_\perp dk^+ \theta(k^+)}{(2\pi)^{3/2}\sqrt{2k^+}}\left[a(\boldsymbol{k})e^{-ik\cdot x} + a^\dagger(\boldsymbol{k})e^{ik\cdot x}\right], \tag{10}$$

where $k\cdot x = \frac{1}{2}(k^- x^+ + k^+ x^-) - \boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp$ with $k^- = \frac{k_\perp^2 + m_s^2}{k^+}$, and $\boldsymbol{k} \equiv (k^+, \boldsymbol{k}_\perp)$. The θ function restricts k^+ to positive values. Note that

$$i\partial^+ e^{-ik\cdot x} = k^+ e^{-ik\cdot x}. \tag{11}$$

The value of x^+ that appears in Eq. (10) can be set to zero, but only after taking necessary derivatives.

Deriving the equal x^+ commutation relations for the fields is a somewhat obscure procedure [11], but the result can be stated in terms of familiar commutation relations:

$$[a(\boldsymbol{k}), a^\dagger(\boldsymbol{k}')] = \delta(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp)\delta(k^+ - k'^+) \tag{12}$$

with $[a(\boldsymbol{k}), a(\boldsymbol{k}')] = 0$.

The next step is compute the Hamiltonian P^- for this system. The conserved energy-momentum tensor is given in terms of the Lagrangian:

$$T_\phi^{\mu\nu} = -g^{\mu\nu}\mathcal{L}_\phi + \frac{\partial \mathcal{L}_\phi}{\partial(\partial_\mu \phi)}\partial^\nu \phi.. \tag{13}$$

This brings us to the question of what is $g^{\mu\nu}$? This is straightforward, although the results (viewed for the first time) can be surprising:

$$g^{+\nu} = g^{0\nu} + g^{3\nu} \tag{14}$$

Thus

$$\begin{aligned} g^{++} &= g^{00} + g^{03} + g^{30} + g^{33} = 1 + 0 + 0 - 1 = 0 \\ g^{ij} &= -\delta_{i,j}(i = 1,2, j = 1,2); \quad g^{+-} = g^{-+} = 2. \end{aligned} \tag{15}$$

Then one finds that

$$T_\phi^{+-} = \frac{1}{2}\boldsymbol{\nabla}_\perp \phi \cdot \boldsymbol{\nabla}_\perp \phi + \frac{1}{2}m_s^2 \phi^2. \tag{16}$$

The term T^{+-} is the density for the operator P^-:

$$P^- = \frac{1}{2}\int d^2x_\perp dx^- T^{+-}. \tag{17}$$

The use of the field expansion (10), along with normal ordering followed by integration leads to the result:

$$P_\phi^- = \int d^2k_\perp dk^+ \theta(k^+) a^\dagger(\boldsymbol{k}) a(\boldsymbol{k}) \frac{k_\perp^2 + m_s^2}{k^+}. \tag{18}$$

One defines a vacuum state $\mid 0\rangle$ such that $a(\boldsymbol{p}) \mid 0\rangle = 0$. Then the creation operators acting on the vacuum give the usual single particle states:

$$P_\phi^- a^\dagger(\boldsymbol{p}) \mid 0\rangle = \frac{p_\perp^2 + m_s^2}{p^+} a^\dagger(\boldsymbol{p}) \mid 0\rangle. \tag{19}$$

The momentum operator P^+ is constructed by integrating T^{++}:

$$P_\phi^+ = \int d^2k_\perp dk^+ \theta(k^+) a^\dagger(\boldsymbol{k}) a(\boldsymbol{k}) k^+. \tag{20}$$

Interactions and Light Front Simplification

Suppose we take the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2) + \lambda\phi^4. \tag{21}$$

The operator ϕ creates or destroys a particles of plus-momenta $k^+ > 0$. Thus a possible term in which $\lambda\phi^4$ term converts the vacuum $\mid 0\rangle$ into a four particle state vanishes by virtue of the conservation of plus-momentum. The vacuum of $p^+ = 0$ can not be connected to four particles, each having a positive k^+. This vanishing simplifies Hamiltonian (x^+-ordered perturbation) calculations.

Free Dirac Field

Consider the Lagrangian

$$\mathcal{L}_\psi = \bar{\psi}(\gamma^\mu \frac{i}{2} \overleftrightarrow{\partial}_\mu - M)\psi, \tag{22}$$

and its equation of motion:

$$(i\gamma^\mu\partial_\mu - M)\,\psi = 0. \tag{23}$$

A fermion has spin 1/2, so there can only be two independent degrees of freedom. The standard Dirac spinor has four components, so two of these must represent dependent degrees of freedom. In the light front formalism one separates the independent and dependent degrees of freedom by using projection operators: $\Lambda_\pm \equiv \frac{1}{2}\gamma^0\gamma^\pm$. Then the independent field is $\psi_+ = \Lambda_+\psi$ and the dependent one is $\psi_- = \Lambda_-\psi$

The Dirac equation (23) is re-written as

$$\left(\frac{i}{2}\gamma^+\partial^- + \frac{i}{2}\gamma^-\partial^+ + i\boldsymbol{\gamma}_\perp\cdot\boldsymbol{\nabla}_\perp - M\right)\psi = 0. \tag{24}$$

Equations for $\psi_\pm$ can be obtained by multiplying Eq. (24) on the left by $\Lambda_\pm$:

$$\begin{aligned} i\partial^-\psi_+ &= (\boldsymbol{\alpha}_\perp\cdot\frac{\boldsymbol{\nabla}_\perp}{i} + \beta M)\psi_- \\ i\partial^+\psi_- &= (\boldsymbol{\alpha}_\perp\cdot\frac{\boldsymbol{\nabla}_\perp}{i} + \beta M)\psi_+, \end{aligned} \tag{25}$$

so that the equation of motion of ψ_+ becomes

$$i\partial^-\psi_+ = (\boldsymbol{\alpha}_\perp\cdot\frac{\boldsymbol{\nabla}_\perp}{i} + M)\frac{1}{i\partial^+}(\boldsymbol{\alpha}_\perp\cdot\frac{\boldsymbol{\nabla}_\perp}{i} + M)\psi_+. \tag{26}$$

One can make the field expansion and determine the momenta in a manner similar to the previous section. The key results are

$$T^{+-}_\psi = \psi^\dagger_+\left(\alpha_\perp\cdot\frac{\boldsymbol{\nabla}_\perp}{i} + \beta M\right)\frac{1}{i\partial^+}\left(\alpha_\perp\cdot\frac{\boldsymbol{\nabla}_\perp}{i} + \beta M\right)\psi_+, \tag{27}$$

$$P^-_\psi = \sum_\lambda\int d^2p_\perp dp^+\theta(p^+)\frac{p_\perp^2 + M^2}{p^+}\left[b^\dagger(\boldsymbol{p},\lambda)b(\boldsymbol{p},\lambda) + d^\dagger(\boldsymbol{p},\lambda)d(\boldsymbol{p},\lambda)\right], \tag{28}$$

where $b(\boldsymbol{p},\lambda), d(\boldsymbol{p},\lambda)$ are nucleon and anti-nucleon destruction operators.

Free Vector Meson

The formalism for massive vector mesons was worked out by Soper [12] and later by Yan [13] using a different formulation. I generally follow Yan's approach. The formalism is lengthy and detailed in the references, so I only state the minimum. There are three independent degrees of freedom, even though the Lagrangian depends on V^μ and $V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$. These are chosen to be V^+ and V^{+i}. The other terms V^-, V^i, V^{-i} and V^{ij} can be written in terms of V^+ and V^{+i}.

We need a Lagrangian, no matter how bad

It seems to me that one can not do complete dynamical calculations using the light front formalism without specifying some Lagrangian. One starts [8] with $\mathcal{L}$ and derives field equations. These are used to express the dependent degrees of freedom in terms of independent ones. One also uses $\mathcal{L}$ to derive $T^{\mu\nu}$ (as a function of independent degrees of freedom) which is used to obtain the total momentum operators $P^\pm$. It is P^- that acts as a Hamiltonian operator in the light front x^+-ordered perturbation theory.

We start with a Lagrangian containing scalar and vector mesons and nucleons ψ'. This is the minimal Lagrangian for obtaining a caricature of nuclear physics because the exchange of scalar mesons provides a medium range attraction which can bind the nucleons and the exchange of vector mesons provides the short-range repulsion which prevents a collapse. Thus we take

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2) - \frac{1}{4}V^{\mu\nu}V_{\mu\nu} + \frac{m_v^2}{2}V^\mu V_\mu \\ + \bar{\psi}'\left(\gamma^\mu(\frac{i}{2}\overleftrightarrow{\partial}_\mu - g_v\, V_\mu) - M - g_s\phi\right)\psi', \tag{29}$$

with the effects of other mesons included elsewhere and below. The equations of motion are

$$\partial_\mu V^{\mu\nu} + m_v^2 V^\nu = g_v\bar{\psi}'\gamma^\nu\psi' \tag{30}$$

$$\partial_\mu\partial^\mu\phi + m_s^2\phi = -g_s\bar{\psi}'\psi', \tag{31}$$

$$(i\partial^- - g_v V^-)\psi'_+ = (\boldsymbol{\alpha}_\perp\cdot(\boldsymbol{p}_\perp - g_v\boldsymbol{V}_\perp) + \beta(M + g_s\phi))\psi'_- \tag{32}$$

$$(i\partial^+ - g_v V^+)\psi'_- = (\boldsymbol{\alpha}_\perp\cdot(\boldsymbol{p}_\perp - g_v\boldsymbol{V}_\perp) + \beta(M + g_s\phi))\psi'_+. \tag{33}$$

The presence of the interaction term V^+ on the left-hand side of the second equation presents a problem because one can not easily solve for ψ_- in terms of ψ_+. This difficulty is handled by using the Soper-Yan transformation:

$$\psi' = e^{-ig_v\Lambda(x)}\psi, \qquad \partial^+\Lambda = V^+. \tag{34}$$

Using this in Eqs. (32)-(33) leads to the more usable form

$$(i\partial^- - g_v\bar{V}^-)\psi_+ = (\boldsymbol{\alpha}_\perp\cdot(\boldsymbol{p}_\perp - g_v\bar{\boldsymbol{V}}_\perp) + \beta(M + g_s\phi))\psi_- \\ i\partial^+\psi_- = (\boldsymbol{\alpha}_\perp\cdot(\boldsymbol{p}_\perp - g_v\bar{\boldsymbol{V}}_\perp) + \beta(M + g_s\phi))\psi_+. \tag{35}$$

The cost of the transformation is that one gets new terms resulting from taking derivatives of $\Lambda(x)$. One uses $\bar{V}^\mu$ with $\bar{V}^\mu = V^\mu - \frac{1}{\partial^+}\partial^\mu V^+$, and $\bar{V}^\mu$ enters in the nucleon field equations, but V^μ enters in the meson field equations.

NUCLEAR MATTER MEAN FIELD THEORY

The philosophy [14] is that the nucleonic densities which are mesonic sources are large enough to generate a large number of mesons to enable a classical treatment (replacing an operator by an expectation value). In infinite nuclear matter, the volume is taken as infinity so that all positions are equivalent. Thus we make the replacement:

$$g_s\bar{\psi}(x)\psi(x) \to g_s\langle\bar{\psi}(0)\psi(0)\rangle, \quad \phi = \frac{-g_s}{m_s^2}\langle\bar{\psi}(0)\psi(0)\rangle, \tag{36}$$

in which the expectation value is in the ground state, and second part of the equation is obtained from the field equation (31) with a constant source. Similarly $g_v\psi(x)\gamma^\mu\psi(x) \rightarrow g_v\langle\bar{\psi(0)}\gamma^\mu\psi(0)\rangle\delta_{\mu,0}$, in which the notion that there is no special direction in space is used. (The nucleus is taken to be at rest.) Again the source is constant, so that the solution of the field equation (30) is

$$\bar{V}^- = V^- = V^0 = \frac{g_v}{m_v^2}\langle\psi^\dagger(0)\psi(0)\rangle; \quad \bar{V}^{+,i} = 0. \tag{37}$$

Since the potentials entering the light-front Dirac equation (35) are constant, the nucleon modes are plane waves $\psi \sim e^{ik\cdot x}$, and the many-body system is a kind of Fermi gas. The solutions of Eq. (35) are

$$i\partial^-\psi_+ = g_v\bar{V}^-\psi_+ + \frac{k_\perp^2 + (M + g_s\phi)^2}{k^+}\psi_+. \tag{38}$$

Solving the equations (36),(37) and (38) yields a self-consistent solution.

Nuclear Momentum Content

The expectation value of $T^{+\mu}$ is used to obtain the total momentum:

$$P^\mu = \frac{1}{2}\int d^2x_\perp dx^-\langle T^{+\mu}\rangle. \tag{39}$$

The expectation value is constant so that the volume $\Omega = \frac{1}{2}\int d^2x_\perp dx^-$ will enter as a factor. A straightforward evaluation leads to the results

$$\frac{P^-}{\Omega} = m_s^2\phi^2 + \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+\ \frac{k_\perp^2 + (M + g_s\phi)^2}{k^+} \tag{40}$$

$$\frac{P^+}{\Omega} = m_v^2(V^-)^2 + \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+\ k^+. \tag{41}$$

To proceed further one needs to define the Fermi surface F. The use of a transformation $k^+ \equiv \sqrt{(M + g_s\phi)^2 + \vec{k}^2} + k^3 \equiv E(k) + k^3$ to define a new variable k^3 enables one to simplify the integrals. One replaces the integral over k^+ by one over k^3 (including the Jacobian factor $\frac{\partial k^+}{\partial k^3} = \frac{k^+}{E}$) leads to:

$$\int_F d^2k_\perp dk^+ \cdots \equiv \int d^3k\theta(k_F - |\vec{k}|)\cdots. \tag{42}$$

The nuclear energy E is the average of P^+ and P^-: $E \equiv \frac{1}{2}(P^- + P^+)$ and one gets the very same expression as in the original Walecka model. This provides a useful check on the algebra.

There is a potential problem: for nuclear matter in its rest frame we need to have $P^+ = P^- = M_A$. If one looks at the expressions for $P^\pm$ this result does not seem

likely. However, the value of the fermi momentum has not yet been determined. There is one more condition to be satisfied:

$$\left(\frac{\partial(E/A)}{\partial k_F}\right)_\Omega = 0. \tag{43}$$

Satisfying this equation determines k_f and for the value so obtained the values of P^+ and P^- turn are the same.

Thus we see that our light front procedure reproduces standard results for energy and density. We use the parameters of Chin and Walecka [15] $g_v^2 M^2/m_v^2 = 195.9$ and $g_s^2 M^2/m_s^2 = 267.1$ to obtain first numerical results. Then $k_F = 1.42 \quad \mathrm{fm}^{-1}$, the binding energy per nucleon is 15.75 MeV and $M + g_s\phi = 0.56M$. The last number corresponds to a huge attraction that is nearly cancelled by the huge repulsion. Then one may use Eq. (41) to obtain the separate contributions of the vector mesons and nucleons, with spectacular results. Nucleons carry only 65% of the plus-momentum. Thus is much less than the 90% needed to explain the EMC effect for infinite nuclear matter [16]. Furthermore, vector mesons carry 35% of the plus-momentum, which is an amazingly large number.

The distribution of this vector meson plus-momentum is an interesting quantity. The mean fields ϕ, V^μ are constants in space and time. Thus V^- has support only for $k^+ = 0$. The physical interpretation of this is that ∞ number of mesons carry a vanishingly small ϵ of the plus-momentum, but the product is 35%. One can also show [17] that

$$k^+ f_v(k^+) = 0.35 M\delta(k^+). \tag{44}$$

There is an important phenomenological consequence the value $k^+ = 0$ corresponds to $x_{Bj} = 0$ which can not be reached in experiments. This means one can't use the momentum sum rule as a phenomenological tool to analyze deep inelastic scattering data to determine the different contributions to the plus-momentum.

Of course this result is caused by solving a simple model for a simple system with a simple mean field approximation. It is necessary to ask if any of the qualitative features of the present results will persist in more detailed treatments.

MEAN FIELD THEORY FOR FINITE-SIZED NUCLEI

It is important to make calculations for finite nuclei because all laboratory experiments are done for such targets or projectiles. The most basic feature of all of nuclear physics is that the shell model is able to explain the magic numbers. Rotational invariance causes the $2j+1$ degeneracy of the single particle orbitals, and full occupation leads to increased binding. But light front dynamics does not make rotational invariance manifest because the different components of the spatial variable are treated differently: $x^-, \boldsymbol{x}_\perp$. However, the final results must respect rotational invariance. Therefore, the challenge of making successful calculations of the properties of finite nuclei is important to us.

Let's discuss, in a general way, how it is that we will be able to find spectra which have the correct number of degenerate states. Suppose we try to determine eigenstates of a LF Hamiltonian by means of a variational calculation. Simply minimizing the LF energy leads to nonsensical results because $P^- = M_A^2/P^+$. One can easily reach zero energy by letting P^+ be infinite. This is not a problem if one is able to use a Fock space basis in which the total plus and $\perp$ momentum of each component are fixed. But in calculations involving many particles, the Fock state approach cannot be used in practical calculations. One needs to find a sensible variational procedure. One such is to perform a constrained variation, in which the total LF momentum is fixed by including a Lagrange multiplier term proportional to the total momentum in the LF Hamiltonian. We minimize the expectation value of P^+ subject to the condition that the expectation values of P^- and P^+ are equal. This is the same as minimizing the expectation value of the average of P^- and P^+.

The need to include the plus-momentum along with the minus momentum can be seen in a simple example. Consider a nucleus of A nucleons of momentum $P_A^+ = M_A$, $\boldsymbol{P}_{A\perp} = 0$, which consists of a nucleon of momentum $(p^+, \boldsymbol{p}_\perp)$, and a residual $(A-1)$ nucleon system which must have momentum $(P_A^+ - p^+, -\boldsymbol{p}_\perp)$. The kinetic energy K is given by the expression

$$K = \frac{p_\perp^2 + M^2}{p^+} + \frac{p_\perp^2 + M_{A-1}^2}{P_A^+ - p^+}. \tag{45}$$

In the second expression, one is tempted to neglect the term p^+ in comparison with $P_A^+ \approx M_A$. This would be a mistake. Instead make the expansion

$$\begin{aligned} K &\approx \frac{p_\perp^2 + M^2}{p^+} + \frac{M_{A-1}^2}{P_A^+}\left(1 + \frac{p^+}{P_A^+}\right) \\ &\approx \frac{p_\perp^2 + M^2}{p^+} + p^+ + M_{A-1}, \end{aligned} \tag{46}$$

because for large A, $M_{A-1}^2/P_A^2 \approx 1$. For free particles, of ordinary three momentum $\boldsymbol{p}$ one has $E^2(p) = \boldsymbol{p}^2 + m^2$ and $p^+ = E(p) + p^3$, so that

$$K \approx \frac{(E^2(p) - (p^3)^2)}{E(p) + p^3} + E(p) + p^3 + M_{A-1} = 2E(p) + M_{A-1}. \tag{47}$$

We see that K depends only on the magnitude of a three-momentum and rotational invariance is restored. The physical mechanism of this restoration is the inclusion of the recoil kinetic energy of the residual nucleus.

Results

The formalism is described in recent papers [9], so I simply summarize the results. If our solutions are to have any relevance, they should respect rotational invariance.

The success in achieving this is examined in Tables I and II of [9] which give our results for the spectra of ^{16}O and ^{40}Ca, respectively. Scalar and vector meson parameters are taken from Horowitz and Serot [18], and we have assumed isospin symmetry. We see that the $J_z = \pm 1/2$ spectrum contains the eigenvalues of all states, since all states must have a $J_z = \pm 1/2$ component. Furthermore, the essential feature that the expected degeneracies among states with different values of J_z are reproduced numerically.

The obtained eigenvalues of the nucleon mode equation are essentially the same as the single particle energies of the ET formalism, to within the expected numerical accuracy of our program. This equality is not mandated by spherical symmetry alone because the solutions in the equal-time framework have non-vanishing components with negative values of p^+. Table III of [9] gives the contributions to the total P^+ momentum from the nucleons, scalar mesons, and vector mesons for ^{16}O, ^{40}Ca, and ^{80}Zr, as well as the nuclear matter limit. The vector mesons carry approximately 30% of the nuclear plus-momentum. The technical reason for the difference with the scalar mesons (which have negligible effect) is that the evaluation of $a^\dagger(\boldsymbol{k}, \omega) a(\boldsymbol{k}, \omega)$ counts vector mesons "in the air" and the resulting expression contains polarization vectors that give a factor of $\frac{1}{k^+}$ which enhances the distribution of vector mesons of low k^+. The results for the vector meson distribution are shown in Fig. 2 of [9]. As the size of the nucleus increases the enhancement of the distribution at lower values of k^+ becomes more evident.

Lepton-nucleus deep inelastic scattering

It is worthwhile to see how the present results are related to lepton-nucleus deep inelastic scattering experiments. We find that the nucleons carry only about 70% of the plus-momentum. The use of our f_N in standard convolution formulae lead to a reduction in the nuclear structure function that is far too large ($\sim$95% is needed [3]) to account for the reduction observed [3] in the vicinity of $x \sim 0.5$. The reason for this is that the quantity $M + g_s\phi$ acts as a nucleon effective mass of about 670 MeV, which is very small. A similar difficulty occurs in the (e, e') reaction [19] when the mean field theory is used for the initial and final states. The use of a small effective mass and a large vector potential enables a simple reproduction of the nuclear spin orbit force [14,18]. Furthermore, the use of other Lagrangians [21,22] will lead to improved results. We also expect that including effects beyond the mean field would lead to a significant effective tensor coupling of the isoscalar vector meson [20], and to an increased value of the effective mass. Such effects are incorporated in Brueckner theory, and a light-front version [10] could be applied to finite nuclei with better success in reproducing the data. This is discussed in the next sections.

CORRELATED INFINITE NUCLEAR MATTER

The first step is to derive a light front version of the nucleon-nucleon interaction. This is most easily done within the framework of the one boson exchange approximation. The formalism and philosophy are discussed in [8], and the calculation is discussed in [10]. The nucleon-nucleon potential $V(NN)$ describes phase shifts reasonably well. The corresponding density is $\mathcal{V}(NN)$. The basic Lagrangian density contains a free nucleon term $\mathcal{L}_0(N)$, a free meson term $\mathcal{L}_0(\text{mesons})$ and an interaction term $\mathcal{L}_I(N, \text{mesons})$ but does not contain $\mathcal{V}(NN)$. Thus one adds this term and subtracts it:

$$\mathcal{L} = \mathcal{L}_0(N) - \mathcal{V}(NN) + \mathcal{L}_{\text{m}} \tag{48}$$

$$\mathcal{L}_{\text{m}} = \mathcal{L}_I(N, \text{mesons}) + \mathcal{L}_0(\text{mesons}) + \mathcal{V}(NN). \tag{49}$$

We use the term $\mathcal{L}_0(N) - \mathcal{V}(NN)$ to obtain a first solution $\mid \Phi\rangle$ to the many-body problem. The term $\mathcal{L}_{\text{m}}$ accounts for mesonic content of Fock space, and we present [10] a scheme to incorporate the effects of $\mathcal{L}_{\text{m}}$ and calculate the full wave function $\mid \Psi\rangle$. Our procedure allows us to assess whether or not $\mathcal{V}(NN)$ has been chosen well. If it has, the effects of $\mathcal{L}_{\text{m}}$ can be treated perturbatively.

Solving for $\mid \Phi\rangle$ is no easy task –it demands a separate non-perturbative treatment. One introduces a mean field U_{MF} which acts on single nucleons.

$$\mathcal{L}_0(N) - \mathcal{V}(NN) = \mathcal{L}_0(N) - U_{MF} + (U_{MF} - \mathcal{V}(NN)) \,. \tag{50}$$

The operator U_{MF} is chosen to minimize the effects of $\langle\Psi|U_{MF} - \mathcal{V}(NN)|\Psi\rangle$. There is a well-known procedure, called Brueckner theory, which is used to determine U_{MF}. In schematic terms:

$$U_{MF} \sim G \times \rho, \tag{51}$$

in which G is a nucleon-nucleon scattering matrix, as modified by the Pauli principle, ρ is the nuclear density, and the $\times$ represents a convolution.

The result [10] is a rather complete theory in which the full wave function $|\Psi\rangle$ includes the effects of both NN correlations and explicit mesons.

Results

The trivial nature of the vacuum in the light front formalism was exploited in deriving [10] the necessary equations. Applying our light front OBEP, the nuclear matter saturation properties are reasonably well reproduced. The binding energy per nucleon is 14.71 MeV with a value of k_F of 1.35 fm^{-1}. This is good considering that we have no three-body force. The computed value of the compressibility, 180 MeV, is smaller than that of alternative relativistic approaches to nuclear matter in which the compressibility usually comes out too large. The replacement

of meson degrees of freedom by a NN interaction was shown to be a reasonable approximation, and that the formalism allows one to calculate corrections to this approximation in a well-organized manner. The mesonic Fock space components of the nuclear wave function are studied we find that there are about 0.05 excess pions per nucleon.

The magnitudes of the scalar and vector potentials are far smaller than found in the mean field approximation. Our first calculation neglected the influence of two-particle-two-hole states to obtain an approximate version of $f(k^+)$ the nucleons carry 81% (as opposed to the 65% of mean field theory) of the nuclear plus momentum. This is a vast improvement in the description of nuclear deep inelastic scattering as the minimum value of the ratio F_{2A}/F_{2N} is increased by a factor of twenty towards the data. This is not enough to provide a satisfactory description, but it is an excellent start. I am optimistic about future results because including nucleons with momentum greater than k_F can be expected to substantially increase the computed ratio F_{2A}/F_{2N} [10].

Let me discuss the observational aspects, concentrating on the experimental information about the nuclear pionic content. The Drell-Yan experiment on nuclear targets [5] showed no enhancement of nuclear pions within an error of about 5%-10% for their heaviest target. Understanding this result is an important challenge to the understanding of nuclear dynamics [6]. Here we have a good description of nuclear dynamics, and our 5% enhancement is consistent, within errors, with the Drell-Yan data.

SUMMARY

The light front approach has been applied, within the mean field approximation, to both infinite and finite nuclear matter. Furthermore, LF studies of πN and NN scattering have been made. This is input to LF calculations of correlated nucleons in infinite nuclear matter. One can use light front dynamics to compute nuclear energies, wave functions and the experimentally observable plus-momentum distributions for a wide variety of Lagrangians. There are indications that the computed quantities will ultimately be in good agreement with experiment. The use of light front dynamics in nuclear physics is only in its infancy, but it seems to be a tool that can be used for any problem in high energy nuclear physics.

ACKNOWLEDGMENTS

These lectures are based on work performed in collaboration with P.G. Blunden, M. Burkardt, and R. Machleidt.

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